

Presentation at bit.ly/1p9Q7Re

Lessons Learned from Doing Number Talks

MaTHink 2016

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RUSD staff development specialists

Big Lessons Learned

1. **Prepare** by doing the math, understanding strategies, and representing student thinking
2. Great **formative** assessment tool
3. Students may need support for grade-level work - **representational tools** can help
4. Making **connections** with other math instruction

Key Understandings

- Number Talks as **formative assessments** of student thinking
- Leverage the **symbiotic relationship** between Number Talks and fluencies, computation, representations, and student engagement in the Mathematical Practices.
- **Explicitly connect** Number Talks to other important mathematics instruction

Arithmetic algorithms are remarkable tools; they are reliable and efficient... The trouble is that their very compactness 'hides the meaning and complexity of the steps involved.'

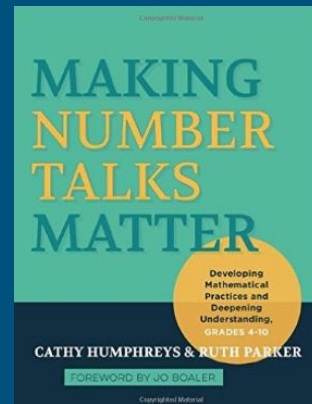
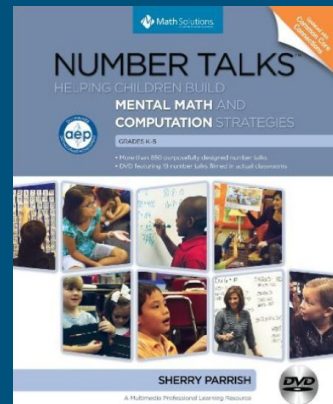
Hyman Bass in Humphreys & Parker (2015)

1. Preparation

Strategies, Properties, & Base-10

Representing student thinking

Using Representational Tools



Mathematics Framework

*for California Public Schools:
Kindergarten Through Grade Twelve*

Single Problem or String?

Language - Vocabulary

Strategies

Student Thinking

Mathematical Representations

Questions

Extensions & Next Steps

Melanie Maxwell

Number Talk Lesson Planning Template (adapted from Georgia Department of Education)

Grade Level: _____ Fluency Target: _____ Strategy focus: _____

Number Talk Problem String(s)/Series	Possible Strategies and Method of Recording	Questions to Students

Academic Language	Next Steps	Extension

Samples of Completed Number Talks & Planning

37-18

37-18
+3 -3
40-15=25
← 15 37 → 40
← 37 → 19

25
-6
19

37-18
35+2
35-20=15
15+4
19

37-18
23-7
-18
21
-18
19

38
-18
20

37-18
37-20=17
+2
19

Easier Prob.; Adjust

18 → 20 → 30 → 37
Add up

12x25

10x25=250
2x25=50
300

25
x12
50
250
300

12x2=240
tens place +
12x5=60
300

	20	5	
10	200	50	
2	40	10	=300

6x(2x25)
6x50=300

25
125
...
12 times

4x25=100
4x25=100
4x25=100
300

Number 130

- How many do you see?
- How do you see it?
- Explain how the dot pattern changes.

3x3=6

5 and 1 more is 6

First Talks - Easing into Expectations

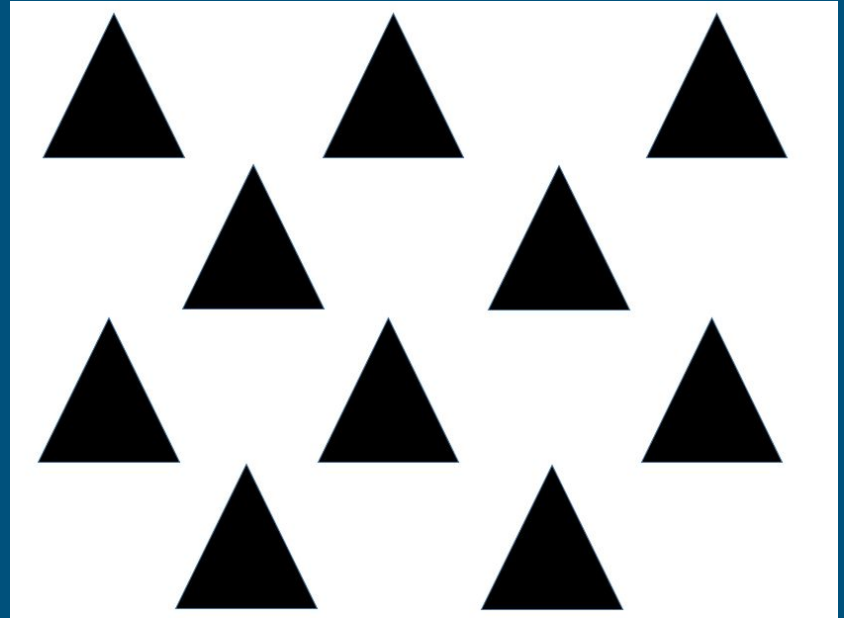
student thinking

explicit & precise explanations

vocabulary - concepts

expose relationships

comparisons



2. Formative Assessment

- Number Talk Shock

Using properties and base ten understanding to compose and decompose

- Make tens and multiples of ten for addition and subtraction
- Using place value and partial products for multiplication and division

3. Representations

Hundred Charts

Number Lines

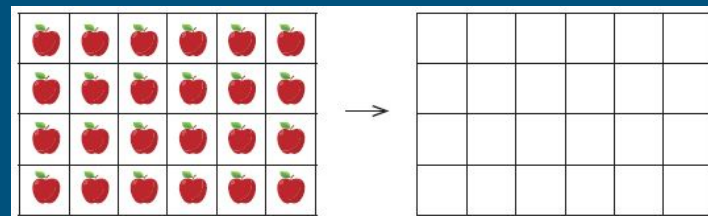
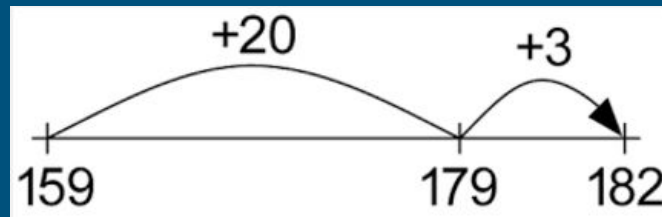
Arrays

Area Models

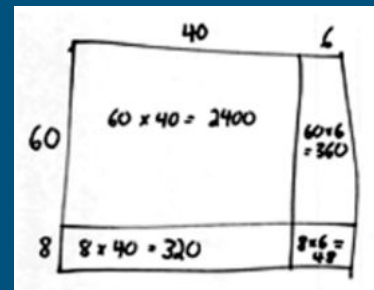
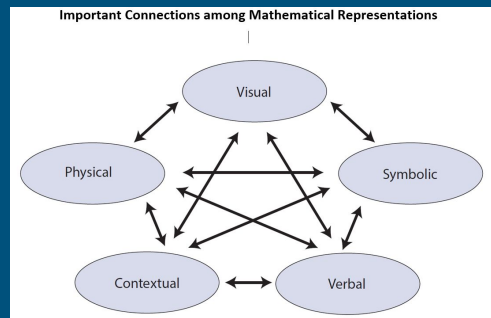
Symbolic

Diagrams

Hundreds Chart									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



$$\begin{array}{r}
 12 \times 15 = \\
 10 \times 15 = 150 \\
 + 2 \times 15 = 30 \\
 \hline
 180
 \end{array}$$



Making 10s

$$3 + 2 + 7$$

—
"Three and two is five,"

$$3 + 2 = 5$$

Then (counting on using fingers),

— "6, 7, 8, 9, 10, 11, 12"

$$\underline{3 + 2 + 7}$$

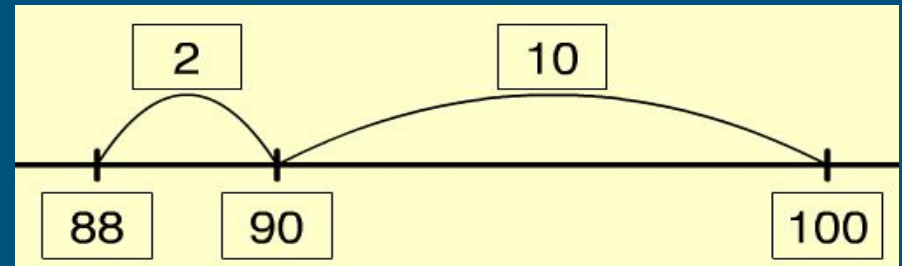
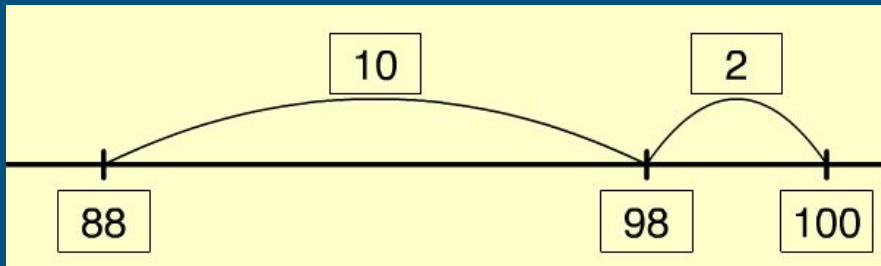
$$3 + 7 = 10$$

$$10 + 2 = 12$$

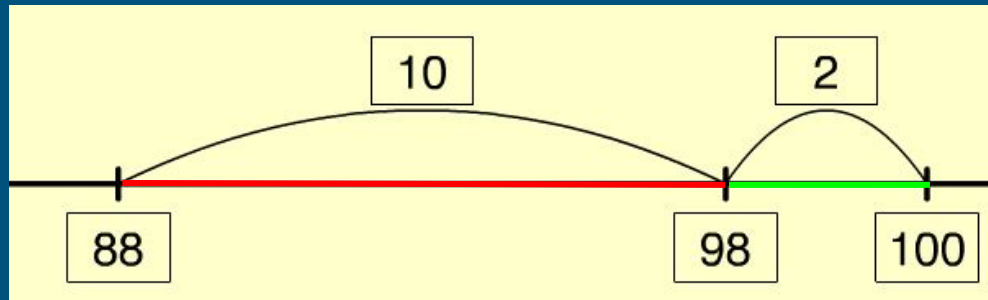
Efficient Strategies

$$\begin{array}{r} \text{---} \\ 88 + 12 = \\ \quad \quad \quad \overset{1}{88} \\ \quad \quad \quad +12 \\ \hline \quad \quad \quad 100 \end{array}$$

120 Chart									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120



Representations Supporting Base-Ten Thinking and Algorithms



81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

$$\underline{88} + \underline{12}$$

$$80 + 10 = 90 \quad 9 \text{ tens}$$

$$8 + 2 = \underline{10} + 1 \text{ ten}$$

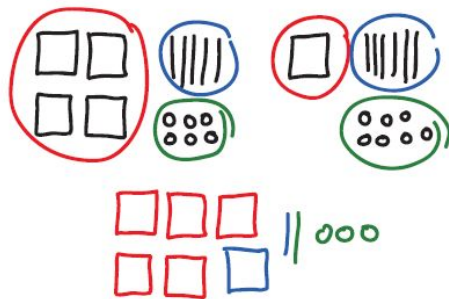
$$100$$

2nd Grade

Examples

2.NBT.7

Addition Method 1: In this written addition method, all partial sums are recorded underneath the addition bar. Addition is performed from left to right in this example, but students can also work from right to left. In the accompanying drawing, it is clear that hundreds are added to hundreds, tens to tens, and ones to ones, which are eventually grouped into larger units where possible to represent the total, 623.



$$\begin{array}{r} 4 \ 5 \ 6 \\ + 1 \ 6 \ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \ 5 \ 6 \\ + 1 \ 6 \ 7 \\ \hline 5 \ 0 \ 0 \end{array}$$

$$\begin{array}{r} 4 \ 5 \ 6 \\ + 1 \ 6 \ 7 \\ \hline 5 \ 0 \ 0 \\ 1 \ 1 \ 0 \end{array}$$

$$\begin{array}{r} 4 \ 5 \ 6 \\ + 1 \ 6 \ 7 \\ \hline 5 \ 0 \ 0 \\ 1 \ 1 \ 0 \\ \hline 1 \ 3 \\ \hline 6 \ 2 \ 3 \end{array}$$

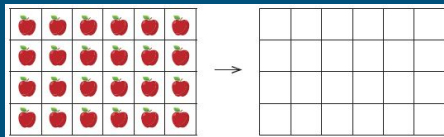


$$\begin{array}{l} 4 + 4 + 4 = 12 \\ 3 + 3 + 3 + 3 = 12 \end{array}$$



$$\begin{array}{l} 5 + 5 + 5 + 5 = 20 \\ 4 + 4 + 4 + 4 + 4 = 20 \end{array}$$

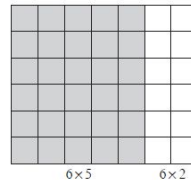
3rd Grade



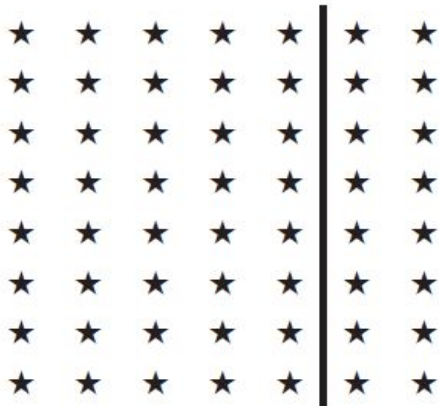
$$7 \times 8 = (5 + 2) \times 8$$

$$8 \times 7 = (4 + 4) \times 7 =$$

Students use area models to represent the distributive property in mathematical reasoning. For example, the area of a 6×7 figure can be determined by finding the area of a 6×5 figure and a 6×2 figure and adding the two sums.



I see that I can arrange the 7 columns into a group of 5 columns and a group of 2 columns.



I know that the 5×8 array gives me 40 and the 2×8 array gives me 16. So altogether I have $5 \times 8 + 2 \times 8 = 40 + 16 = 56$ stars.

I see that I can arrange the 8 rows of stars into 2 groups of 4 rows.



I know that each new 4×7 array gives me 28 stars, so altogether I have $4 \times 7 + 4 \times 7 = 28 + 28 = 56$ stars.

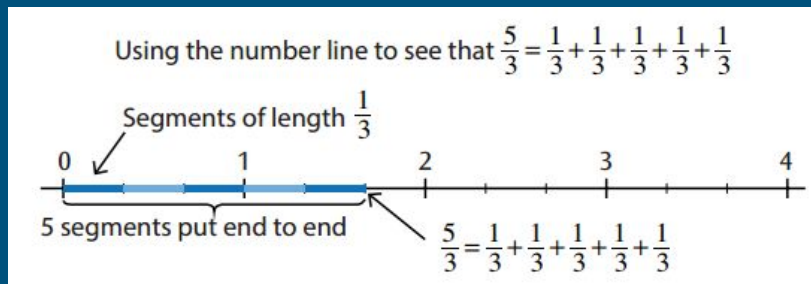
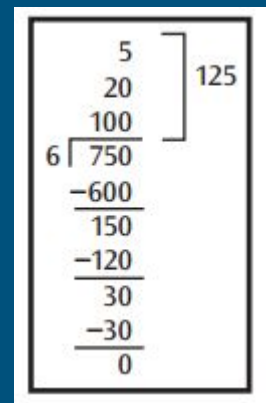
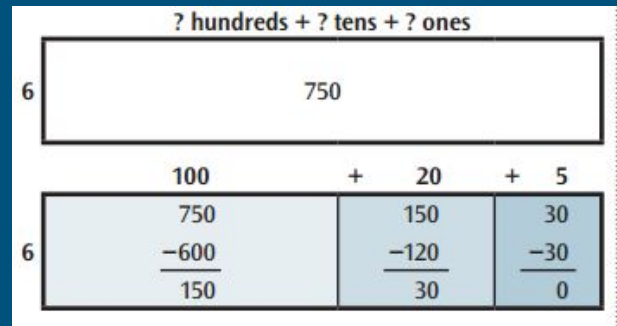
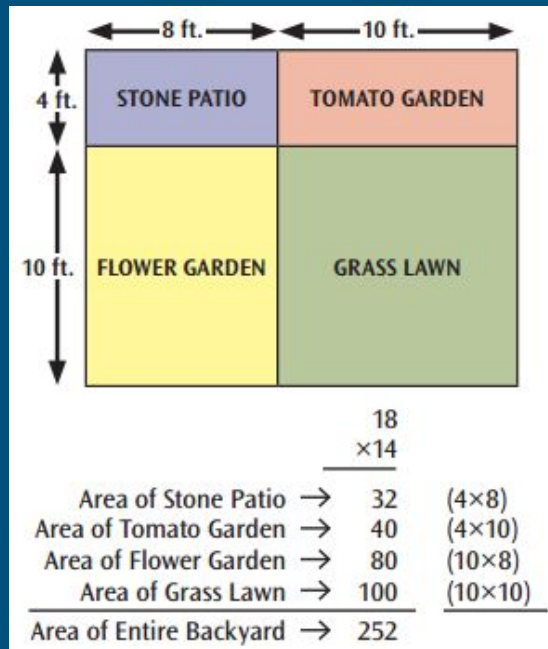
4th grade

Computation

Conceptual

Representation

Algorithm

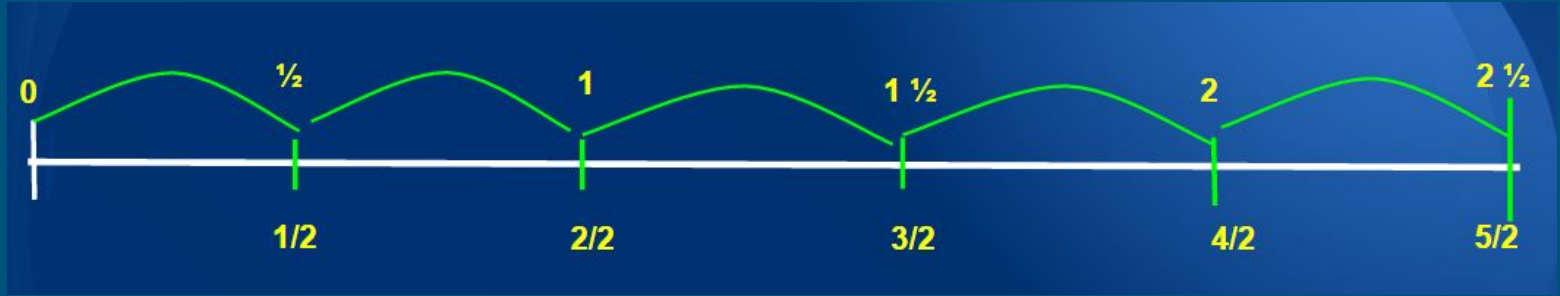


Fraction Multiplication (4th & 5th Grade)

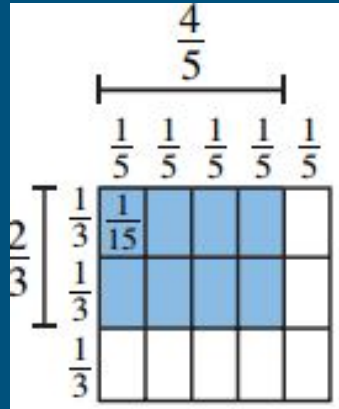
$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$5 \times \frac{1}{2}$$

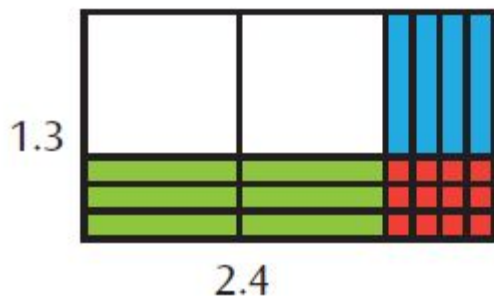
$$2\frac{1}{2} \div 5$$



$$\frac{2}{3} \times \frac{4}{5}$$

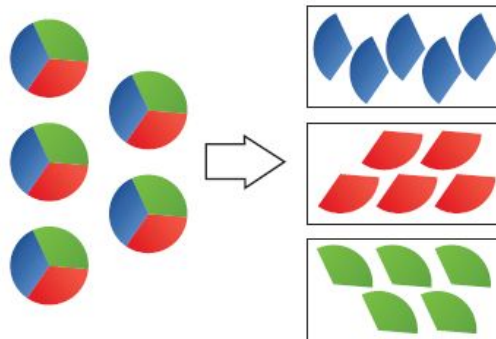


5th Multiplication and Division



$$\begin{array}{r} 2.4 \\ \times 1.3 \\ \hline .12 \\ .60 \\ .40 \\ + 2.00 \\ \hline 3.12 \end{array}$$

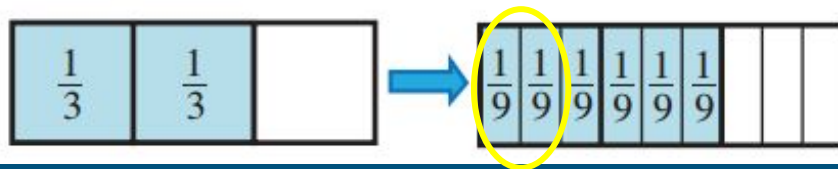
Divide 5 objects into three equal shares, showing that $5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$.



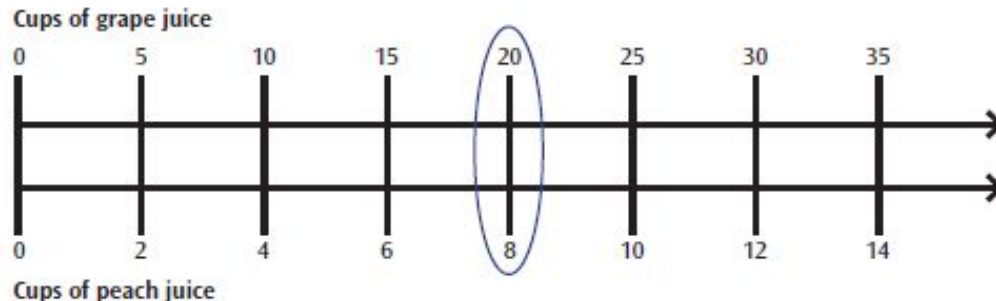
6th Grade

1. Three people share $\frac{2}{3}$ of a pound of watermelon. How much watermelon does each person get?

Solution: This problem can be represented by $\frac{2}{3} \div 3$. To solve it, students might represent the watermelon with a diagram such as the one below. There are two $\frac{1}{3}$ -pound pieces represented in the picture. Students can see that $\frac{1}{3}$ divided among three people is $\frac{1}{9}$. Since there are 2 such pieces, each person receives $\frac{2}{9}$ of a pound of watermelon.



Using a Double Number Line Diagram: "I set up a double number line, with cups of grape juice on the top and cups of peach juice on the bottom. When I count up to 8 cups of peach juice, I see that this brings me to 20 cups of grape juice."



4. Connections to Other Math Instruction

- Fluency (including with algorithms)
- Explaining our thinking
- Investigations
- Direct Instruction

Fluencies

How numbers are
composed and
decomposed*

*Parrish, p.38

FLUENCY

California's Common Core State Standards for Mathematics (K–6) set expectations for fluency in computations using the standard algorithm (e.g., “*Fluently* add and subtract multi-digit whole numbers using the standard algorithm” [4.NBT.4▲]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word *fluent* is used in the standards to mean “reasonably fast and accurate” and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies.

Grade	Required Fluency
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 ¹ Add/subtract within 100 (pencil and paper)
3	Multiply/divide within 100 ² Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations

3 Phases of Fluency

Phase 2

“Deriving answers using reason strategies based on known facts”

FIGURE 1

Students who learn multiplication facts through traditional approaches generally do not retain the facts because the method attempts to move students from phase 1 directly to phase 3 of Baroody's (2006) three developmental phases.

Phases of basic fact mastery (Baroody 2006)

Phase 1: Modeling and/or counting to find the answer

- Solving 6×4 by drawing 6 groups of 4 dots and skip counting the dots



Phase 2: Deriving answers using reasoning strategies based on known facts

- Solving 6×4 by thinking $5 \times 4 = 20$ and adding one more group of 4



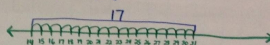
Phase 3: Mastery (efficient production of answers)

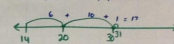
- Knowing that $6 \times 4 = 24$

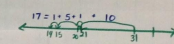
Strategy Charts

Subtraction Strategies

31-14

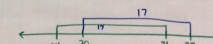
- Adding Up 
(kept track on my fingers)
- Add Up in chunks

$$\begin{array}{r} 14 + 6 = 20 \\ + 10 \\ \hline 30 + 1 = 31 \end{array}$$

- Removal

$$\begin{array}{r} 31 - 10 = 21 \\ - 1 \\ \hline 20 - 5 = 15 \\ - 1 \\ \hline 14 \end{array}$$

- Keeping a Constant Difference

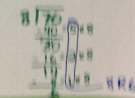
$$\begin{array}{r} 31 + 6 = 37 \\ - 14 + 6 = 20 \\ \hline 20 \end{array}$$

Make this a friendly number (17)



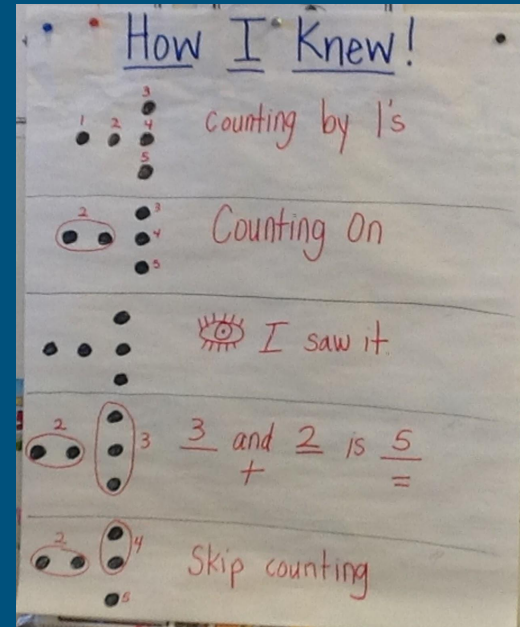
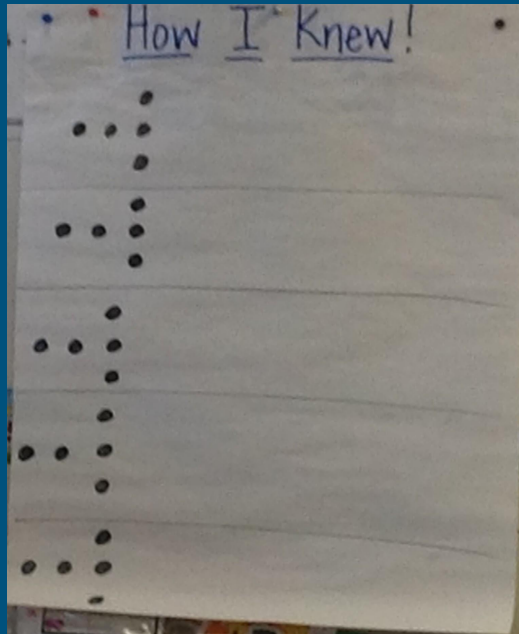
Division Strategies

70 ÷ 8

- Skip Count 8, 16, 24, 32, 40, 48, 56, 64
 Groups: 1 2 3 4 5 6 7 8
 $70 - 64 = 6$ (8R6)
- Repeated Subtraction
 $70 - 8 = 62$; $62 - 8 = 54$; $54 - 8 = 46$
 $46 - 8 = 38$; $38 - 8 = 30$; $30 - 8 = 22$
 $22 - 8 = 14$; $14 - 8 = 6$ (8R6)
- Think Multiplication
 $8 \times \underline{\quad} = 70$ $8 \times 8 = 64$ $\frac{70}{64} = 8 \frac{6}{8}$
 (8R6)
- Array $8 \overline{)70} \rightarrow 8 \overline{)70} \begin{array}{l} 8 \times 8 = 64 \\ 70 - 64 = 6 \end{array}$ (8R6)
- Subtract Parts 

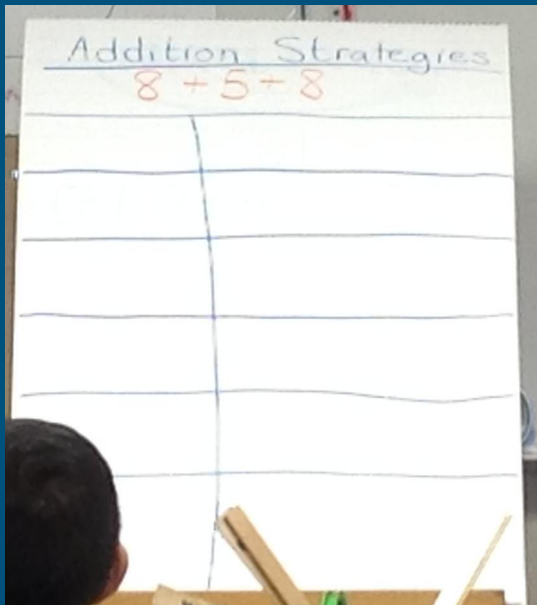
One Way to Create a Strategy Chart

Plan, Pencil, Coordinate, Side Notes as needed.



One Way to Create a Strategy Chart

Plan, Pencil, Coordinate, Side Notes as needed.



Addition Strategies	
$8 + 5 + 8$ 21	
Count all	
Counting On	$\{8\}$ $9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21$
Added in chunks	$8 + 5 = 13$ $13 + 8 = 21$
Break apart	$13 + 4 = 17$ $17 + 2 = 19$ $19 + 2 = 21$
Know and use <u>Doubles</u>	$8 + 8 = 16$ $16 + 5 = 21$
Make 10	$5 + 5 = 10$ $10 + 3 = 13$ $13 + 8 = 21$

$$\begin{array}{r} 8 + 5 + 8 \\ \wedge \quad \quad \quad \wedge \\ 5 + 3 \quad \quad 5 + 3 \end{array}$$

Break apart

$$3 + 3 = 6$$

Doubles

$$5 + 5 = 10$$

make 10

$$\begin{array}{r} + 5 \\ 15 \\ 15 + 6 = 21 \end{array}$$

One Way to Create a Strategy Chart

Plan, Pencil, Coordinate, Side Notes as needed.

Multiplication Strategies example $24 \times 4 = 96$	
Repeat Addition	$(24 + 24) + (24 + 24)$ 48 + 48 96
Use Landmarks Friendly Numbers	$24 \rightarrow 25$ Compatible $4 \times 25 = 100$ $- 4 \times 1 = -4$ 96
Partial Products	$4 \times 4 = 16$ $20 \times 4 = 80$ 96 24×4 $\begin{array}{r} 20 \times 4 \\ 4 \times 4 \end{array}$ 24×4 $\begin{array}{r} 10 \times 4 \\ 14 \times 4 \end{array}$ $10 \times 4 = 40$ $14 \times 4 = 56$ 96
Standard Algorithm	$\begin{array}{r} 24 \\ \times 4 \\ \hline 96 \end{array}$
	$\begin{array}{r} 24 \times 4 \\ \wedge \\ 12 \times 2 \times 4 \\ 12 \times 8 \end{array}$

(Main idea)

$$4 \times 24 =$$

$$24 + 24 + 24 + 24$$

$$24^4 =$$

$$24 \times 24 \times 24 \times 24$$

Practicing Strategies & Representations

- Frayer Model
- Investigations
 - “So and so did ...”
 - Does the method work? Why or why not?
 - Does it ever work? Does it always work?
- Decisions:
 - Monitoring your Number Talk time
 - When and how to incorporate things that rise in Number Talks into instruction and vice versa

Summary: Number Talks Rock!

Apply a variety of strategies

Monitor and reflect on process

Justification as a fundamental aspect of math

Make and investigate conjectures

Select and use various types of reasoning and proof

Organize and consolidate thinking through communication

Communicate thinking clearly and accurately

Language of mathematics to express ideas precisely

Recognize connections among mathematical ideas

Create and use representations to organize, record, and communicate mathematical ideas

Mathematical
Practices are
Supported

Summary: Number Talks Rock!

Mathematical

Conceptual Understanding- Operations, relations

Procedural Fluency- Compute flexibly, accurately, efficiently appropriately

Proficiencies and

Strategic Competence- Represent and solve

Adaptive Reasoning - Logical thought, reflection, explanation, justification

Productive Disposition- See math as sensible, worthwhile, and belief in one's own efficacy and as a *doer of mathematics*

are Supported

Today's Key Points about Number Talks

Implications for MY classroom

Formative assessment

Students use multiple representations
for conceptual understanding, solving,
and justifying reasoning

Supports a variety of opportunities for
students to engage in the
Mathematical Practice Standards

Supports fluency in operations

Planning and Next Steps

Melanie Marwan

Number Talk Lesson Planning Template (adapted from Georgia Department of Education)

Grade Level:

Fluency Target:

Strategy focus:

Number Talk Problem String(s)/Series	Possible Strategies and Method of Recording	Questions to Students

Academic Language	Next Steps	Extension

Self-Reflection

- What did the students know? How do they know it?
- Where could you have incorporated representations, possibly by asking, “May I show your strategy using a _____?”
- What vocabulary did you use? What vocabulary did the students use?
- Were student ideas expressed effectively and precisely?



MaTHink 2016 Mini Conference

California Standards for Mathematics

Deepen your knowledge of California Standards, and learn how they will affect learning and teaching in the next few years. We will provide active sessions that will help make this information personal and applicable to your classroom. Topics include:

- Content Support
- Strategies for All Learners
- Assessment
- Discourse in the Mathematics Classroom
- Science, Technology, Engineering, and Mathematics (STEM)
- Community Outreach
- Math Coaching
- Mindsets

